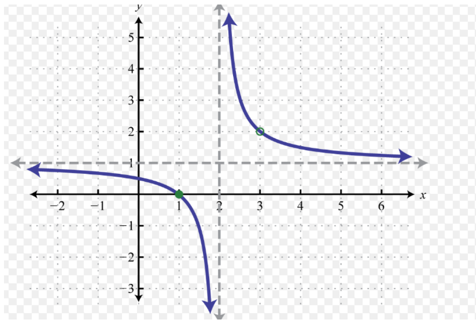
2.6-2.7 Rational Functions

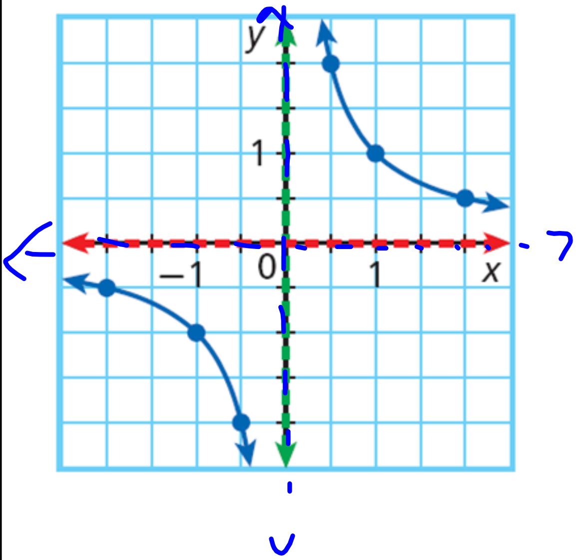
Rational Function : a function whose rule can be written as a ratio of two polynomials. Its graph is a hyperbola, which has two separate branches.

$$f(x) = \frac{p(x)}{q(x)}$$

where p(x) and q(x) are polynomial functions, and $q(x) \neq 0$



Parent Rational Function



$$f(x) = \frac{1}{x}$$

Domain: $x \neq 0$

Range: $y \neq 0$

Vertical Asymptote(s): $x = 0$

Horizontal Asymptote(s): $y = 0$

Transformations of Simple Rational Functions

Simple Rational Function: $f(x) = \dfrac{a}{x-h} + k$

$|a|$ ⟶ vertical stretch or compression factor
$a < 0$ ⟶ reflection across the x-axis

k ⟶ vertical translation
down for $k < 0$; up for $k > 0$

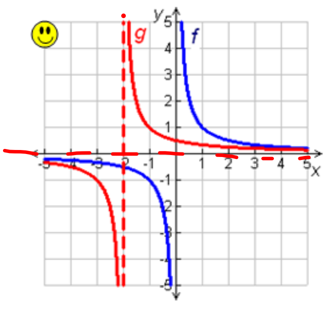
$$f(x) = \frac{a}{x-h} + k$$

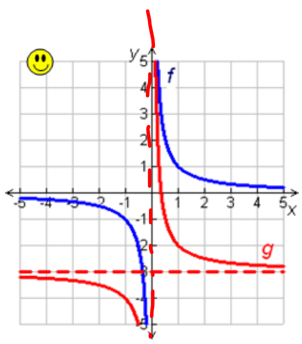
h ⟶ horizontal translation
left for $h < 0$; right for $h > 0$

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

A. $g(x) = \dfrac{1}{x+2}$ shifts left 2

B. $g(x) = \dfrac{1}{x} - 3$ down 3





What do you notice about the location of the vertical and horizontal asymptotes?

$x = h$ ⟵ Vertical Asymptote

$y = k$ ⟵ H.A.

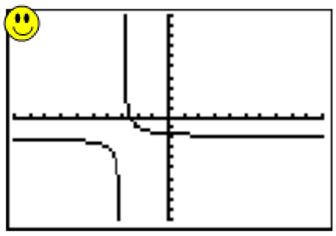
Rational Functions

For a rational function of the form $f(x) = \frac{a}{x-h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$.

Identify the domain, range and all asymptotes of the given functions.

1.) $f(x) = \frac{1}{x+3} - 2$



Domain: $x \neq -3$

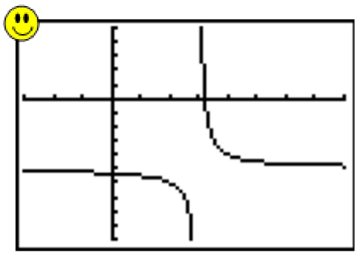
Range: $y \neq -2$

Vertical Asymptote(s): $x = -3$

Horizontal Asymptote(s): $y = -2$

Identify the domain, range and all asymptotes of the given functions.

2.) $f(x) = \frac{1}{x-3} - 5$



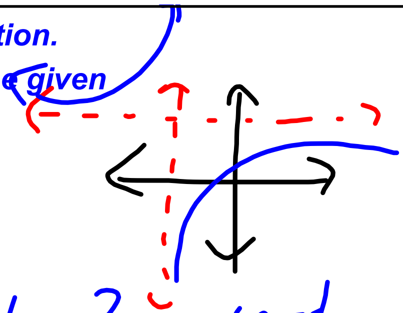
Domain: $x \neq 3$

Range: $y \neq -5$

Vertical Asymptote(s): $x = 3$

Horizontal Asymptote(s): $y = -5$

Describe the transformations of each rational function. Identify the domain, range and all asymptotes of the given function. *Draw a sketch of the graph to help.



3.) $f(x)=\dfrac{-2}{x+2}+3$

Transformations: up 3, left 2, vert. stretch by factor of 2

reflection in x-axis

Vertical Asymptote(s):

$x = -2$

Domain:

$x \neq -2$

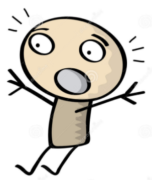
Horizontal Asymptote(s):

$y = 3$

Range:

$y \neq 3$

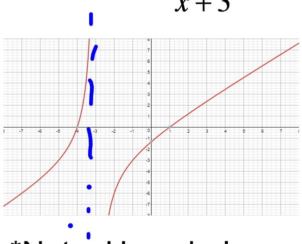
Not all Rational Functions are Simple!



General Form: $f(x)=\dfrac{p(x)}{q(x)};\ q(x)\neq 0$

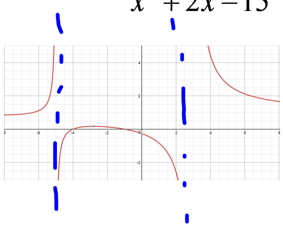
Examples:

a.) $f(x)=\dfrac{x^2+3x-4}{x+3}$



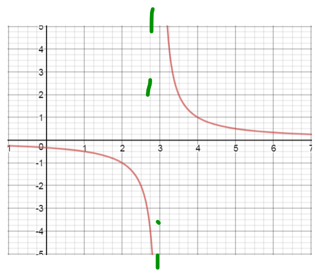
*Not a Hyperbola

b.) $f(x)=\dfrac{x^2+5x+4}{x^2+2x-15}$



*Not a Hyperbola

c.) $f(x)=\dfrac{x+4}{x^2+x-12}$



Identifying Vertical Asymptotes of any Rational Function:

Find the value of x for which the denominator = 0.

a.) $f(x)=\frac{x^2+3x-4}{x+3}$

V.A. $x=-3$

b.) $f(x)=\frac{x^2+5x+4}{x^2+2x-15} = \frac{x^2+5x+4}{(x+5)(x-3)}$

V.A. $x=-5,\ x=3$

c.) $f(x)=\frac{x+4}{x^2+x-12} = \frac{\cancel{x+4}}{\cancel{(x+4)}(x-3)} = \frac{1}{x-3}$

V.A. $x=3$

Hmmm....Something's up with example c....

A **hole** is an omitted point in a graph.

Holes in Graphs **Rational Functions**

If a rational function has the same factor $x-b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x=b$, unless the line $x=b$ is a vertical asymptote.

You try...

Identify any holes and vertical asymptotes for each rational function.

4.) $f(x)=\frac{x^2-25}{x^2+12x+35} = \frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x+7)}$

hole @ $x=-5$

V.A. $x=-7$

5.) $f(x)=\frac{x+4}{2x^2+5x-3} = \frac{(x+4)}{(2x-1)(x+3)}$

V.A. $x=1/2,\ x=-3$

no holes

6.) $f(x)=\frac{x^2-9}{3x^2+8x-3} = \frac{\cancel{(x+3)}(x-3)}{(3x-1)\cancel{(x+3)}}$

hole @ $x=-3$

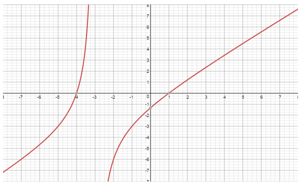
VA: $x=1/3$

Identifying Horizontal Asymptotes of any Rational Function:

The existence and location of a horizontal asymptote depends on the degree of the polynomials that make up the rational function.

Examples:

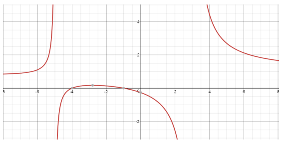
$a.)\ f(x)=\dfrac{x^2+3x-4}{x+3}$



Horizontal Asymptote?

no

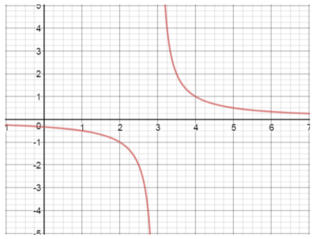
$b.)\ f(x)=\dfrac{x^2+5x+4}{x^2+2x-15}$



Horizontal Asymptote?

yes $y=1$

$c.)\ f(x)=\dfrac{x+4}{x^2+x-12}$ yes $y=0$



Horizontal Asymptote?

Identifying Horizontal Asymptotes of any Rational Function:

$$f(x)=\frac{N(x)}{D(x)};\ D(x)\neq 0$$

If *N* degree > *D* degree: H.A.: Does not exist

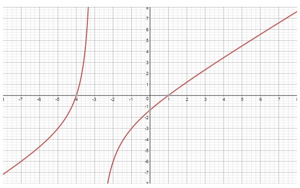
If *N* degree = *D* degree: H.A.: *y* = ratio of leading coefficients

$$f(x)=\frac{\text{leading coefficient of } N}{\text{leading coefficient of } D}$$

If *N* degree < *D* degree: H.A.: *y* = 0

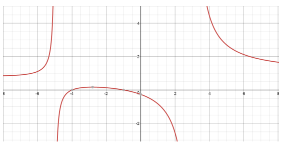
Examples:

$a.)\ f(x)=\dfrac{x^2+3x-4}{x+3}$



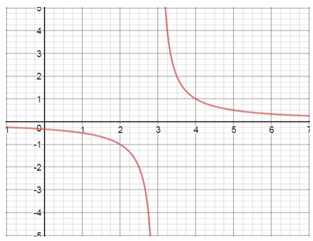
Horizontal Asymptote?

$b.)\ f(x)=\dfrac{x^2+5x+4}{x^2+2x-15}$



Horizontal Asymptote?

$c.)\ f(x)=\dfrac{x+4}{x^2+x-12}$



Horizontal Asymptote?

You try...
Identify the horizontal asymptote for each rational function. If there is not a horizontal asymptote, explain why.

7.) $f(x)=\dfrac{x^2-25}{x^2+12x+35}$

8.) $f(x)=\dfrac{x+4}{2x^2+5x-3}$

9.) $f(x)=\dfrac{x^2-9}{3x^2+8x-3}$

Putting it all together..
Find all vertical/horizontal asymptotes and identify any holes, if they exist.

10.) $f(x)=\dfrac{x^2-16}{x-4}$

11.) $f(x)=\dfrac{x^2}{x^2-9}$

12.) $f(x)=\dfrac{5x-25}{x^2-x-20}$