5.3 Solving Trigonometric Equations

Solving Trigonometric Equations

- Isolate trig function. Factor or apply other quadratic strategies first, if necessary. Make use of fundamental trigonometric identities as needed.
- Depending on the period of isolated trig function, identify all solutions on the interval $[0, 2\pi)$ or $[0, \pi)$. → $\sin, \cos, \csc, \sec$ $\tan, \cot$
- State the general solution (which accounts for ALL solutions).
- List all solutions on interval $[0, 2\pi)$.
- Check your solutions!!!

Example #1	$2\cos x - 1 = 0$ $2A - 1 = 0$

$$2\cos x = 1$$
$$\cos x = \frac{1}{2}$$

General solution: $x = \frac{\pi}{3} + 2n\pi$, $x = \frac{5\pi}{3} + 2n\pi$

List of solutions on $[0, 2\pi)$: $\frac{\pi}{3}, \frac{5\pi}{3}$

Example #2	$3\tan^2 x - 1 = 0$

$$3\tan^2 x = 1$$
$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$
$$\tan x = \pm\frac{\sqrt{1}}{\sqrt{3}}$$
$$\tan x = \pm\frac{\sqrt{3}}{3}$$

General solution: $\frac{\pi}{6} + n\pi$, $\frac{5\pi}{6} + n\pi$

List: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Example #3 $2\sin^2 x + 3\sin x + 1 = 0$ $2s^2 + 3s + 1 = 0$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$2\sin x + 1 = 0 \qquad\qquad \sin x + 1 = 0$

$\sin x = -\tfrac{1}{2} \qquad\qquad \sin x = -1$

General: $\dfrac{7\pi}{6} + 2n\pi,\quad \dfrac{11\pi}{6} + 2n\pi,\quad \dfrac{3\pi}{2} + 2n\pi$

List: $\dfrac{7\pi}{6},\ \dfrac{11\pi}{6},\ \dfrac{3\pi}{2}$

Example #4 $\csc x - 2 = 0$

$$\csc x = 2$$

$$\sin x = \tfrac{1}{2}$$

General: $\pi/6 + 2n\pi,\quad \dfrac{5\pi}{6} + 2n\pi$

List: $\dfrac{\pi}{6},\ \dfrac{5\pi}{6}$

Example #5　　　　　$2\cos^2 x - 1 = 0$

$$2\cos^2 x = 1$$
$$\cos^2 x = \tfrac{1}{2}$$
$$\cos x = \pm\sqrt{\tfrac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

General: $\frac{\pi}{4} + 2n\pi,\ \frac{3\pi}{4} + 2n\pi,\ \frac{5\pi}{4} + 2n\pi,\ \frac{7\pi}{4} + 2n\pi$

List: $\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

Example #6　　　　$\sec x \csc x - 2\csc x = 0$　　　$SC - 2C = 0$

$$\csc x(\sec x - 2) = 0$$

$\csc x = 0$　　　　　　　$\sec x = 2$

$\sin x = und$　　　　　　$\cos x = \tfrac{1}{2}$

General: $\frac{\pi}{3} + 2n\pi,\ \frac{5\pi}{3} + 2n\pi$

List: $\frac{\pi}{3},\ \frac{5\pi}{3}$

Example #7 $2\sec^2 x + \tan^2 x - 3 = 0$

$$2\sec^2 x + (\sec^2 x - 1) - 3 = 0$$

$$3\sec^2 x - 4 = 0$$
$$3\sec^2 x = 4$$
$$\sec^2 x = 4/3$$
$$\sqrt{\cos^2 x} = \sqrt{3/4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$

General: $\frac{\pi}{6} + 2n\pi,$ $\frac{11\pi}{6} + 2n\pi,$ $\frac{5\pi}{6} + 2n\pi,$ $\frac{7\pi}{6} + 2n\pi$

List: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Example #8 $(\cos x + 1)^2 = (\sin x)^2$ $\cos x + 1 = \sin x$

$$(\cos x + 1)(\cos x + 1)$$
$$\cos^2 x + 2\cos x + 1 = \sin^2 x$$
$$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$$
$$+\cos^2 x \qquad\qquad -1 \cdot 1 + \cos^2 x$$

$$2\cos^2 x + 2\cos x = 0$$
$$2\cos x(\cos x + 1) = 0$$
$$2\cos x = 0 \qquad \cos x + 1 = 0 \qquad \cos x = -1$$
$$\cos x = 0$$

General: $\frac{\pi}{2} + 2n\pi,$ $\frac{3\pi}{2} + 2n\pi,$ $\pi + 2n\pi$

List: $\frac{\pi}{2}, \pi$