Warm Up

FLASHBACK

In 1 - 2, find the midpoint of *A* and *B*.

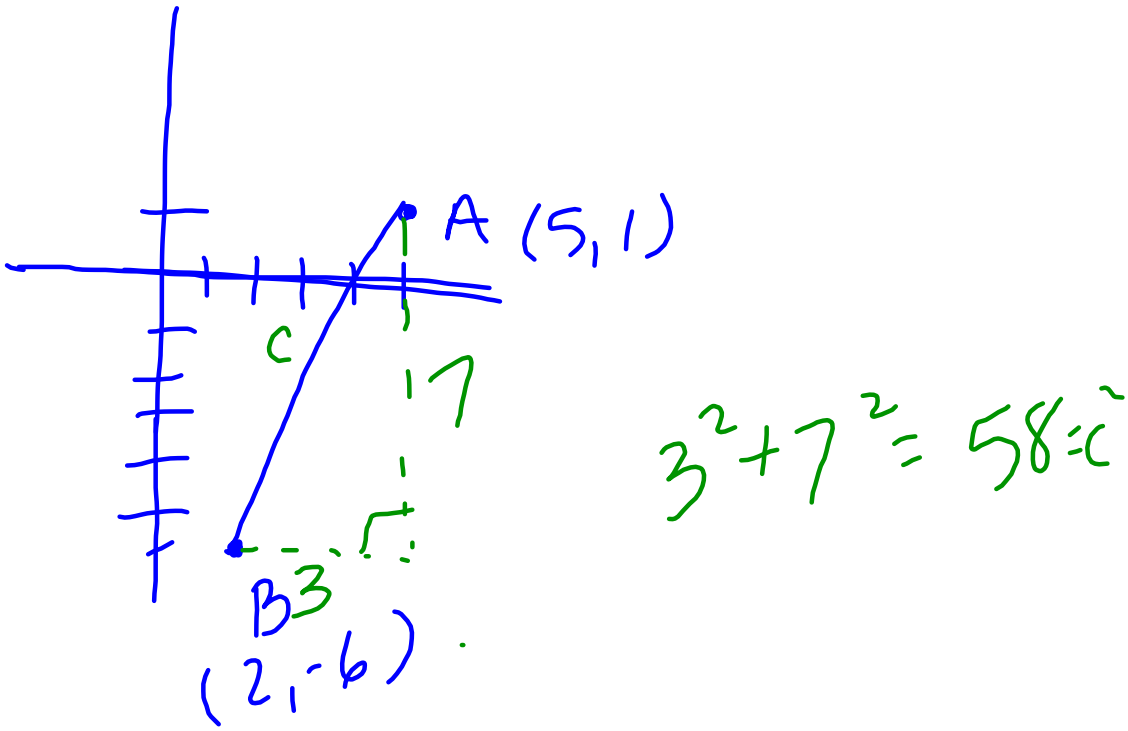
1. ***A*(3, -2); *B*(7, -2)** (5, -2)
2. ***A*(5, 1); *B*(2, -6)** (7/2, -5/2)

Midpoint Formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

In 3 - 4, find the length of line segment *AB*.

3. ***A*(3, -2); *B*(7, -2)** AB = 4
4. ***A*(5, 1); *B*(2, -6)** $AB = \sqrt{58}$

distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



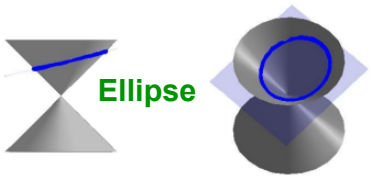
$3^2+7^2=58=c^2$

Chapter 9: Introduction to Conics

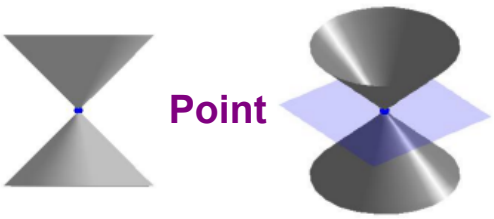
Pre-Requisites: Completing the Square and Circles

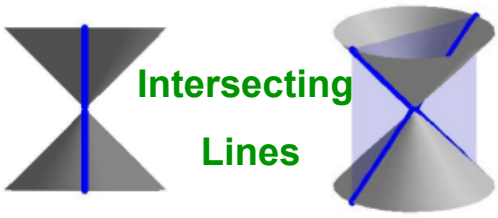
A **conic section** is the intersection of a plane and a double-napped cone.

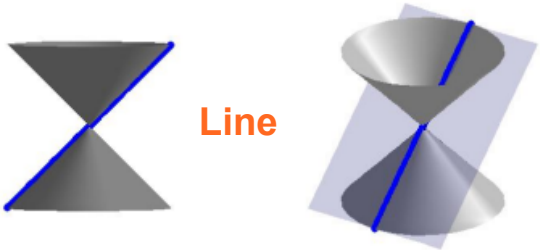
There are four basic conics:



If the slicing plane contains the vertex of the cone, we get the so-called **degenerate conics**: a point, a line, or two intersecting lines.

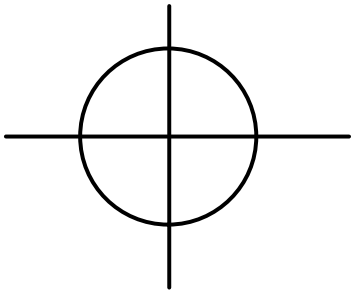






The equation of a circle:

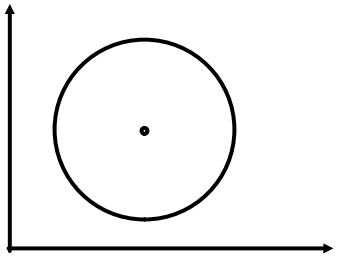
with radius r and center at the origin:



a.) $r = 6$ with C(0, 0)

$x^2 + y^2 = 36$

with radius r and center at (h, k):



(h, k)

b.) r = 7 with C(0, -9)

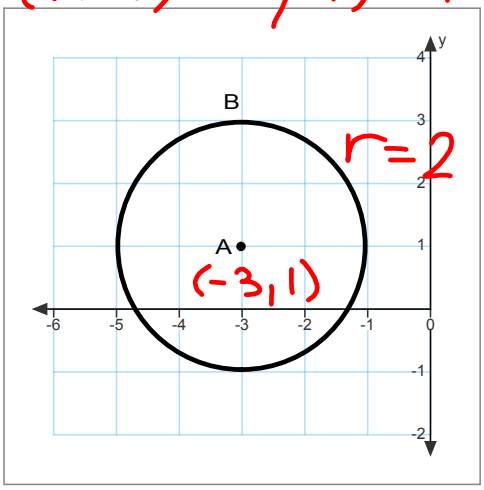
$x^2 + (y+9)^2 = 49$

c.) r = 6 with C(-3, 4)

$(x+3)^2 + (y-4)^2 = 36$

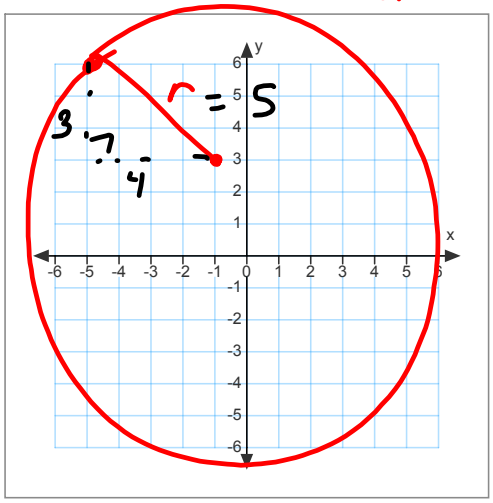
1.) Write the equation of the circle.

$(x+3)^2 + (y-1)^2 = 4$



$(x+1)^2 + (y-3)^2 = 25$

2.) Center (-1, 3) and Point (-5, 6) on circle



Write the equation of the circle. $(x-h)^2+(y-k)^2=r^2$

3.) Endpoints (-11, 13) and Point (4, 5)

of diameter

Midpoint of diameter (center of circle) $= \left(\frac{-11+4}{2}, \frac{13+5}{2}\right) = \left(\frac{-7}{2}, 9\right)$

diameter $= \sqrt{(-11-4)^2+(13-5)^2} = \sqrt{(-15)^2+8^2} = \sqrt{289} = 17$

$r = 17/2$

$$\left(x+\tfrac{7}{2}\right)^2+(y-9)^2=\frac{289}{4}$$

Completing the Square

Solve by completing the square.

4.) $x^2+20x+104=0$

"c" $= \left(\frac{20}{2}\right)^2 =$

$x^2+20x+100 = -104+100$

$\sqrt{(x+10)^2} = \pm\sqrt{-4}$

$x+10 = \pm 2i$

$x = -10 \pm 2i$

Write the Equation of the Circle by Completing the Square

5.) $x^2 + y^2 + 4x - 4y - 17 = 0$ → $(x-h)^2+(y-k)^2=r^2$

$x^2+4x+4 + y^2-4y+4 = 17+4+4$

$(x+2)^2 + (y-2)^2 = 25$

center: $(-2, 2)$

$r = 5$

Write the Equation of the Circle by Completing the Square

6.) $x^2 + y^2 + 8x - 2y - 64 = 0$

$x^2+8x+16 + y^2-2y+1 = 64+16+1$

$(x+4)^2 + (y-1)^2 = 81$

Write the Equation of the Circle by Completing the Square

7.) $x^2 + y^2 + 14x - 12y + 4 = 0$

$x^2+14x+49+y^2-12y+36 = -4+49+36$

$(x+7)^2+(y-6)^2=81$

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Write the Equation of the Circle by Completing the Square

8.) $\dfrac{3x^2 + 3y^2 - 12x + 30y + 75 = 0}{3}$

* leading coefficient must equal 1

$x^2+y^2-4x+10y+25=0$

$x^2-4x+4+y^2+10y+25 = -25+4+25$

$(x-2)^2+(y+5)^2=4$

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Write the Equation of the Circle by Completing the Square

9.) $\dfrac{4x^2+4y^2-5x+8y-2=0}{4}$

$x^2+y^2-\frac{5}{4}x+2y-\frac{1}{2}=0$

$x^2-\frac{5}{4}x+\frac{25}{64}+y^2+2y+1=\frac{1}{2}+\frac{25}{64}+1$

$\left(x-\frac{5}{8}\right)^2+(y+1)^2=\frac{121}{64}$

⋮

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Write the Equation of the Circle by Completing the Square

10.) $2x^2+2y^2+6x-8y+12=0$