

Notes: 5.5, Part II $\frac{1}{2}$ Half-Angle Formulas $\frac{1}{2}$

⑧ $\frac{7\pi}{12}$ is half of $\frac{7\pi}{6}$

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{7\pi/6}{2}\right)$$

Not both - must choose \rightarrow $\begin{pmatrix} + \\ - \end{pmatrix}$ $\sqrt{\frac{1 - \cos\frac{7\pi}{6}}{2}} = + \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{7\pi/6}{2}\right) = + \sqrt{\frac{1 + \cos\frac{7\pi}{6}}{2}} = + \sqrt{\frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{2}} = + \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$\tan\left(\frac{7\pi}{12}\right) = - \sqrt{\frac{1 - \cos\frac{7\pi}{6}}{1 + \cos\frac{7\pi}{6}}}$$

$\sin\frac{7\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$$\cos\frac{7\pi}{12} = \cos\left(\frac{7\pi/6}{2}\right)$$

$$= \begin{pmatrix} + \\ - \end{pmatrix} \sqrt{\frac{1 + \cos\frac{7\pi}{6}}{2}}$$

$$= - \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$= - \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}} = - \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$\cos\frac{7\pi}{12} = \frac{-\sqrt{2-\sqrt{3}}}{2}$

$$\begin{aligned} \tan \frac{7\pi}{12} &= \tan\left(\frac{7\pi/6}{2}\right) \\ &= \frac{1 - \cos \frac{7\pi}{6}}{\sin \frac{7\pi}{6}} = \frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{-\frac{1}{2}} \\ &= \frac{2 + \sqrt{3}}{-1} \\ \tan \frac{7\pi}{12} &= -2 - \sqrt{3} \end{aligned}$$

⑨ $\sin u = -\frac{3}{5}$, $\cos u = -\frac{4}{5}$, $\csc u = -\frac{5}{3}$
 $\frac{u}{2}$ is in QII

$$\begin{aligned} \sin\left(\frac{u}{2}\right) &= \sqrt{\frac{1 - \cos u}{2}} \\ &= + \sqrt{\frac{1 + \frac{4}{5}}{2}} = + \sqrt{\frac{9/5}{2}} \\ &= + \sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \\ \cos\left(\frac{u}{2}\right) &= -\sqrt{\frac{1 + \cos u}{2}} \\ &= -\sqrt{\frac{1 + \frac{4}{5}}{2}} = -\sqrt{\frac{9/5}{2}} \\ \tan\left(\frac{u}{2}\right) &= \frac{\sin u}{1 + \cos u} = \frac{-3/5}{1 - 4/5} \\ \tan \frac{u}{2} &= \frac{-3/5}{1/5} = -3 \end{aligned}$$

⑩ verify

$$\tan \frac{x}{2} = \csc x - \cot x$$

$$\frac{1 - \cos x}{\sin x} =$$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} =$$

$$\csc x - \cot x = \csc x - \cot x \quad \checkmark$$

⑫ $\frac{u}{2}$ $u = 225^\circ$

$$\sin(112.5^\circ) = \sin\left(\frac{225^\circ}{2}\right)$$

$$= \sqrt{\frac{1 - \cos 225^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\cos(112.5^\circ) = -\sqrt{\frac{1 + \cos 225^\circ}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$\cos 112.5^\circ = -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\tan 112.5^\circ = \frac{1 - \cos 225^\circ}{\sin 225^\circ} = \frac{1 + \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$$

$$= \frac{2 + \sqrt{2}}{-\sqrt{2}} = \boxed{-\sqrt{2} - 1}$$

U in	$\frac{1}{2}$ is n
QI	QI
QII	QI
$QIII$	QII
QIV	QII