

Pre-Calculus

Notes: 2.6 Graphs of Simple Rational Functions

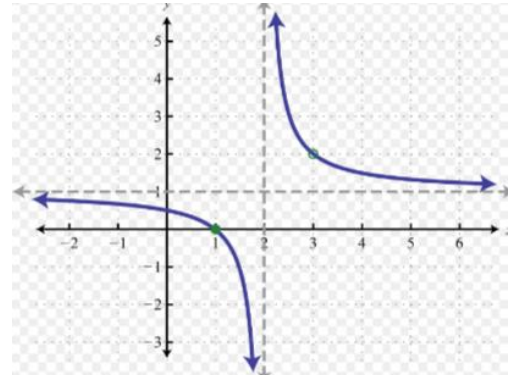
Name \_\_\_\_\_

Date \_\_\_\_\_ Block \_\_\_\_\_

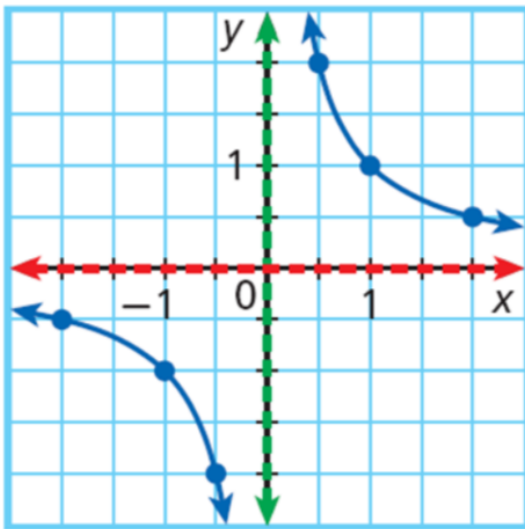
Rational Function: \_\_\_\_\_

$$f(x) = \frac{p(x)}{q(x)}$$

where  $p(x)$  and  $q(x)$  are polynomial functions, and  $q(x) \neq 0$



Parent Rational Function:



$f(x) =$

Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptote(s):

Transformations of Simple Rational Functions

Simple Rational Function:

$|a| \rightarrow$   
 $a < 0 \rightarrow$  reflection across the

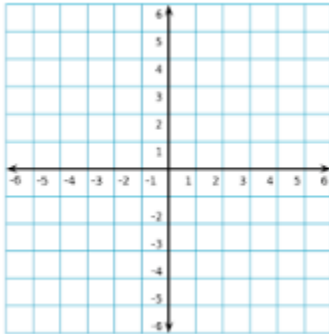
$k \rightarrow$   
 for  $k < 0$ ; for  $k > 0$

$f(x) = \frac{a}{x - h} + k$

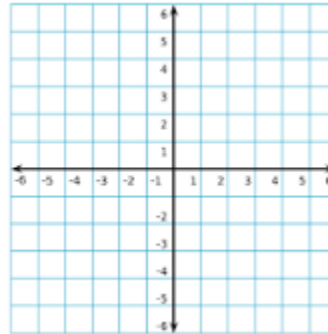
$h \rightarrow$   
 for  $h < 0$ ; for  $h > 0$

Using the graph of  $f(x) = \frac{1}{x}$  as a guide, describe the transformation and sketch the graph of each function.

A.)



B.)



What do you notice about the location of the horizontal and vertical asymptotes?

### Rational Functions

For a rational function of the form  $f(x) = \frac{a}{x - h} + k$ ,

- the graph is a hyperbola.
- there is a vertical asymptote at the line  $x = h$ , and the domain is  $\{x \mid x \neq h\}$ .
- there is a horizontal asymptote at the line  $y = k$ , and the range is  $\{y \mid y \neq k\}$ .

Identify the domain, range and all asymptotes of the given function.

1.)

Domain:

Range:

Vertical Asymptote(s):

Horizontal Asymptote(s):

Identify the domain, range and all asymptotes of the given function.

2.)

Domain:

Range:

Vertical Asymptote(s):

Horizontal Asymptote(s):

Describe the transformations of each rational function. Identify the domain, range and all asymptotes of the given function. \*Draw a sketch of the graph to help.

3.)

Transformations:

Vertical Asymptote(s):

Domain:

Horizontal Asymptote(s):

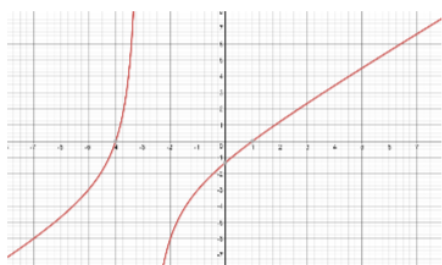
Range:

Not all Rational Functions are Simple!

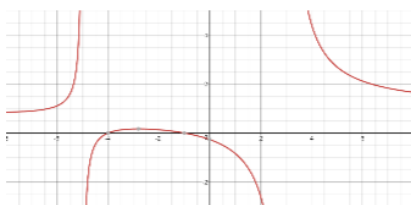
General Form:

Examples:

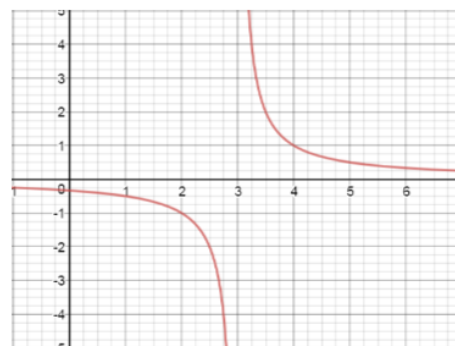
a.)  $f(x) =$



b.)  $f(x) =$



c.)



## Identifying Vertical Asymptotes of Any Rational Function:

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a.)  $f(x) = \frac{x^2 + 3x - 4}{x + 3}$

b.)  $f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$

c.)  $f(x) = \frac{x + 4}{x^2 + x - 12}$



Hmmmm.....

**You try...Identify any holes and vertical asymptotes for each rational function.**

4.)  $f(x) = \frac{x^2 - 25}{x^2 + 12x + 35}$

5.)  $f(x) = \frac{x + 4}{2x^2 + 5x - 3}$

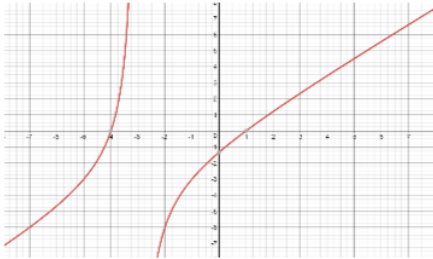
6.)  $f(x) = \frac{x^2 - 9}{3x^2 + 8x - 3}$

**Identifying Horizontal Asymptotes of Any Rational Function:**

\*

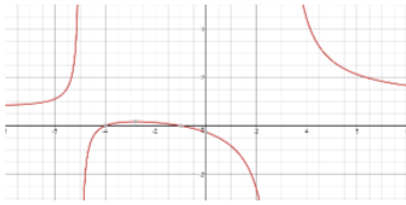
**Examples:**

$$a.) f(x) = \frac{x^2 + 3x - 4}{x + 3}$$



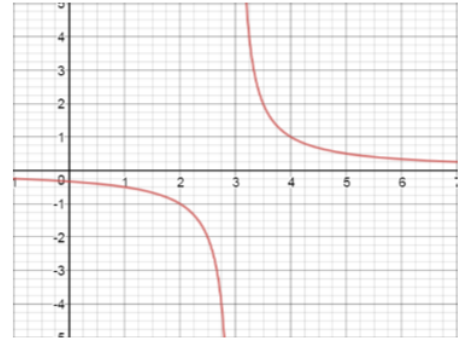
Horizontal Asymptote?

$$b.) f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$$



Horizontal Asymptote?

$$c.) f(x) = \frac{x + 4}{x^2 + x - 12}$$



Horizontal Asymptote?

**Identifying Horizontal Asymptotes of Any Rational Function:**

If  $N$  degree  $>$   $D$  degree: \_\_\_\_\_

If  $N$  degree  $=$   $D$  degree: \_\_\_\_\_

If  $N$  degree  $<$   $D$  degree: \_\_\_\_\_

**You try...**

**Identify the horizontal asymptote for each rational function. If there is not a horizontal asymptote, explain why.**

$$7.) f(x) = \frac{x^2 - 25}{x^2 + 12x + 35}$$

$$8.) f(x) = \frac{x + 4}{2x^2 + 5x - 3}$$

$$9.) f(x) = \frac{x^2 - 9}{3x^2 + 8x - 3}$$

**Putting it all together...**

**Find all vertical/horizontal asymptotes and identify any holes if they exist.**

$$10.) f(x) = \frac{x^2 - 16}{x - 4}$$

$$11.) f(x) = \frac{x^2}{x^2 - 9}$$

$$12.) f(x) = \frac{5x - 25}{x^2 - x - 20}$$