WS 5.1-5.3 Review ANSWER KEY

1. $(\sec^2 x - 1)(\cos^2 x) = \sin^2 x$

$\sec^2 x(\cos^2 x) - \cos^2 x =$

$1 - \cos^2 x =$

$\sin^2 x = \sin^2 x$ ✓

2. $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

$\cot^2 x(\csc^2 x - 1) =$

$\cot^2 x(\cot^2 x) =$

$\cot^4 x = \cot^4 x$ ✓

3. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

$\sec^2 x(1 + \cot^2 x) =$

$\sec^2 x + \sec^2 x \cot^2 x =$

$\sec^2 x + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} =$

$\sec^2 x + \csc^2 x = \sec^2 x + \csc^2 x$ ✓

4. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$

$\frac{\sec x \cos x - \sin^2 x}{\sin x \cos x} =$

$\frac{1 - \sin^2 x}{\sin x \cos x} =$

$\frac{\cos^2 x}{\sin x \cos x} =$

$\frac{\cos x}{\sin x} =$

$\cot x = \cot x$ ✓

5. $$\frac{\cos x}{1+\sin x} + \tan x = \sec x$$

$$\frac{\cos x}{(1+\sin x)} \cdot \frac{(1-\sin x)}{(1-\sin x)} + \frac{\sin x}{\cos x} =$$

$$\frac{\cos x\,(1-\sin x)}{1-\sin^2 x} + \frac{\sin x}{\cos x} =$$

$$\frac{\cos x\,(1-\sin x)}{\cos^2 x} + \frac{\sin x}{\cos x} =$$

$$\frac{1-\sin x + \sin x}{\cos x} =$$

$$\sec x = \sec x \checkmark$$

6. $$\frac{\csc(-x)}{\sec(-x)} = -\cot x$$

$$\frac{-\csc x}{\sec x} =$$

$$\frac{\cos x}{-\sin x} =$$

$$-\cot x = -\cot x \checkmark$$

7. $$\frac{\sec x \sin x}{\tan x + \cot x} = \sin^2 x$$

$$\frac{\frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}}{\frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}} =$$

$$\frac{\frac{\sin^2 x}{\cos x \sin x}}{\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}} =$$

$$\frac{\sin^2 x}{\sin^2 x + \cos^2 x}$$

$$\sin^2 x = \sin^2 x \checkmark$$

8. $\csc^4 x - \cot^4 x = 2\csc^2 x - 1$

$(\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x) =$

$(\csc^2 x + \cot^2 x)\ 1 =$

$\csc^2 x + \csc^2 x - 1 =$

$2\csc^2 x - 1 = 2\csc^2 x - 1$ ✓

9. $\csc^2 x \tan^2 x - 1 = \tan^2 x$

$(1 + \cot^2 x)\tan^2 x - 1 =$

$\tan^2 x + \cot^2 x \tan^2 x - 1 =$

$\tan^2 x + 1 - 1 =$

$\tan^2 x = \tan^2 x$ ✓

10. $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1 - \cos x}$

$\dfrac{(\cos x + 1)}{\sin x (\sin^2 x)} =$

$\dfrac{(\cos x + 1)}{\sin x (1 - \cos^2 x)} =$

$\dfrac{(\cos x + 1)}{\sin x (1 - \cos x)(1 + \cos x)}$

$\dfrac{1}{\sin x (1 - \cos x)} =$

$\dfrac{\csc x}{1 - \cos x} = \dfrac{\csc x}{1 - \cos x}$ ✓

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \frac{\pi}{6} + 2n\pi, \quad \frac{5\pi}{6} + 2n\pi$

$[0, 2\pi)$: $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

12. $\sin x = \sqrt{3} - \sin x$

$2\sin x = \sqrt{3}$

$\sin x = \sqrt{3}/2$

$x = \frac{\pi}{3} + 2n\pi, \quad \frac{2\pi}{3} + 2n\pi$

$[0, 2\pi)$: $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

13. $\tan x + 1 = 0$

$\tan x = -1$

$x = \frac{3\pi}{4} + n\pi$

$[0, 2\pi)$: $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

14. $\frac{1}{2}\sec x - 1 = 0$

$\frac{1}{2}\sec x = 1$

$\sec x = 2$

$\cos x = 1/2$

$x = \frac{\pi}{3} + 2n\pi, \quad \frac{5\pi}{3} + 2n\pi$

$[0, 2\pi)$: $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

15. $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x - 1 = 0$

$-\sin^2 x + \sin x = 0$

$-\sin x (\sin x - 1) = 0$

$-\sin x = 0 \qquad \sin x - 1 = 0$

$\sin x = 0 \qquad \sin x = 1$

$x = 0 + 2n\pi, \ \pi + 2n\pi \qquad x = \frac{\pi}{2} + 2n\pi$

$[0, 2\pi)$: $\left\{0, \frac{\pi}{2}, \pi\right\}$

16. $2\cos 2x - \sqrt{2} = 0$

$\cos 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2n\pi \qquad 2x = \frac{7\pi}{4} + 2n\pi$

$x = \frac{\pi}{8} + n\pi, \quad x = \frac{7\pi}{8} + n\pi$

$\left\{\frac{\pi}{8}, \frac{9\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}\right\}$

$\cos^2 x - 3\cos x + 1 = 0$

$\cos x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$\cos x = \dfrac{3 \pm \sqrt{9-4}}{2}$

$\cos x = \dfrac{3 \pm \sqrt{5}}{2} \approx 2.618$
0.382

$x = \cos^{-1}(2.618) \approx \varnothing$

$x = \cos^{-1}(0.382) \approx 1.179 + 2n\pi$
AND $5.104 + 2n\pi$

$[0, 2\pi): \{1.179, 5.104\}$

18. $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

$3x = \frac{\pi}{3} + n\pi \quad 3x = \frac{2\pi}{3} + n\pi$

$x = \frac{\pi}{9} + \frac{n\pi}{3} \quad x = \frac{2\pi}{9} + \frac{n\pi}{3}$

$[0, 2\pi): \{\frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}\}$.

19.

$\tan^2 x - \tan x - 12 = 0$

$(\tan x - 4)(\tan x + 3) = 0$

$\tan x = 4 \quad \tan x = -3$

$x = 1.326 + n\pi$

$x = -1.249$

$x = 1.893 + n\pi$

$\{1.326, 4.468$
$1.893, 5.034\}$

20. $\sec^2 x + 6\tan x + 4 = 0$

$1 + \tan^2 x + 6\tan x + 4 = 0$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 5)(\tan x + 1) = 0$

$\tan x = -5 \quad \tan x = -1$

$x = 1.769 + n\pi, \quad 2.356 + n\pi$

$[0, 2\pi): \{1.769, 2.356, 4.911, 5.498\}$